Simulation models for mineral processing plants

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ABSTRACT

Simulation is the process of designing a computerized model, of a system or procedure, for the purpose of understanding its behaviour and developing strategies to control its operation. Simulation models are rapidly becoming an effective tool for mineral processing plants. Powerful software and fast hardware are more readily available than a basic understanding of the theory of mathematical probability and applied statistics that process simulation models demand.

Simple spreadsheet software can be used to develop simulation models that take into account any set of conditions at a mineral processing plant. In addition to an unbiased estimate for each variable in the set on which two-and three-product formulas are based, reliable variance estimates for all variables are required. How to estimate variances, effectively and at the lowest possible cost, and how to compute variances for simple functions such as the metal content of a quantity of ore or concentrate, or more complex functions such as the percentage recovery at a mineral processing plant, are the key to useful simulation models.

Introduction

Simulation models for systems and procedures have become one of the most powerful and effective applications for computers[2,6]. Simulations are applied to a wide range of scientific and engineering disciplines. Generally, a model is designed to investigate the effects of changes in and interactions between a set of variables before a system is in operation or a procedure is implemented, by simulating weeks, months, and even years of operation, in a few minutes of computer time.

Along with practical and useful applications always come abuse and misuse. For a simulation process cannot be more reliable than the methodology that was applied to design the model, to estimate variables and variances, to check performance, and to verify validity. A mineral processing plant is a dynamic system that changes continuously as a function of time but within probabilistic constraints that can be estimated by applying statistical tools and techniques to the variables and variances that interact in the system.

A model should be designed to simulate not only the set of variables but also the variance for each variable. Models in which constant variances are used to generate normally distributed random numbers, are less realistic than models in which the variances are simulated first, and then used to simulate the variables.

A deterministic model may apply to one subset of variables while a stochastic model may be more suitable to describe others.

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A system is deterministic if its present state is completely determined by a previous state, and stochastic if its present state cannot be predicted from a previous state with a high degree of certainty. Variables that interact in mineral processing plants often exhibit deterministic and stochastic behaviours so that correlations between past and present states may display varying degree of significance. Dependencies between variables should be identified and taken into account.

Simulation models are based on mathematical probability and applied statistics[1,3,5]. One of the most important postulates in applied statistics is the Central Limit Theorem. In this application of the theorem it dictates that the degree of precision with which the behaviour of a dynamic system can be determined, increases with the number of measurements that are performed to interrogate it. The cost differential between a small number of real measurements and a very large set of simulated measurements essentially accounts for the power and popularity of simulation models.

Any computer with spreadsheet software is a suitable tool to simulate the operation of a mineral processing plant. The advantage of a spreadsheet is that the simulation model can easily be modified to reflect changes in process parameters, or better understanding of its operation. The simulation model can only be meaningful if it takes into account all possible interactions between a set of variables, and all possible effects of dependencies between subsets of variables. Spreadsheet software is suprisingly efficient and flexible for all types of simulation models.

The variance for the variable that contributes most to the system's intrinsic uncertainty is the limiting factor for the precision of a simulation model which implies that simulations can also be used to identify the most dominant variable in a system or procedure. How to obtain reliable variance estimates at the lowest possible cost, how to optimize simulations most effectively, and how to compute variances of functions such as contained metal or recovery in percent are the essence of simulation models for mineral processing plants.

Variances for Variables

Realistic variance estimates for variables and functions are essential for effective simulation models of mineral processing plants. The sampling theory for bulk materials is based on the additive property of variances[3,5]. Typical applications are variances for metal grades and contents. For example, the total variance for the measurement chain that estimates metal grades of a quantity of ore, concentrate or tailing is the sum of the sampling variance, the preparation variance and the analytical variance. By contrast, the difference between variances is only an unbiased estimate for the variance of an intermediate stage in a measurement chain if their F-ratio is statistically significant.

The Central Limit theorem dictates that the variance for the mean of a set of elements (primary or secondary increments, subsamples or test samples, or replicate assays) is equal to the variance between single elements divided by the number of elements

TABLE 3. Variables and variances for copper and gold contents

Parameter	Symbol	Cu in mt	Au in kg
Wet mass in mt	Mw	10 000	25 000
Variance in mt²	var(Mw)	25 600	275 625
Moisture factor	MF	0.968	0.975
Variance	var(MF)	$0.0256 * 10^{-4}$	$0.011,664 * 10^{-4}$
Copper factor	AF	0.0053	
Variance	var(AF)	$0.0176 * 10^{-6}$	
Gold grade in kg/mt	a		0.0032
Variance in (kg/mt)²	var(a)		$0.013,271 * 10^{-6}$
Copper content in mt	Cu	51.304	
Variance in mt²	var(Cu)	2.3302	
Gold content in kg	Au		78.08
Variance in kg²	var(au)		10.5970

0.1462 = 25 077 mt.

Based on a variance of 275 625 mt², and a rounded z-value of 2.0, the 95% CI for the wet mass of 25 000 mt is: $2.0 * \sqrt{275\,625} = \pm 1050$ mt for a 95% CR range from: 25 000 − 1050 = 23 950 mt up to: 25 000 + 1050 = 26 050 mt. Randomly generated wet masses of 24 429 mt and 25 077 mt fall within this 95% CR for the observed wet mass of 25 000 mt so that each appears to be an unbiased estimate for the unknown true mass of this quantity of crushed ore.

Not only the wet mass of 25 000 mt but also its variance of 275 625 mt² can be generated by simulation. After all, variances too are stochastic variables that are precise within predictable probability limits. Any set of normally distributed random numbers can be used to simulate a variance but a set of twelve will be used to simulate variances in this example.

The validity of simulation models for variances with the "@RAND" function in spreadsheet software is verified by applying statistical tests. Bartlett's chi-squared test is applied to check a set of variances for homogeneity while Fisher's F-test is applied to check a pair of variances for compatibility.

For example, twelve variance estimates that were each based on twelve normally distributed random numbers resulted in an average variance of 229 111 mt² for the wet mass of 25 000 mt. The F-ratio of: 275 625/229 111 = 1.20 between a calculated variance of 275 625 mt² and a simulated variance of 229.111 mt² is below the tabulated values of F0.95;∞;132 = 1.24 and F0.99;∞;132 = 1.36. Hence, the simulated variance is compatible with the estimated variance which implies that the "@RAND" pseudo-random number generator is unbiased for small sequences. Infinite degrees of freedom were applied to this variance estimate of 275 625 mt² but in practice a variance can only be estimated with finite degrees of freedom. If this variance were based on a set of twenty measurements, then the tabulated values would be F0.95;19;132 = 1.65 and F0.99;19;132 = 2.02.

Homogeneity of Variances for Contained Metal

The wet mass, metal grade and moisture content of a quantity of ore, and the variances for this set of variables, were used to compute the variances for copper and gold contents. Table 3 lists all variables and variances that were used to compute the variances for contained metals.

These variables and variances for wet mass, moisture content and metal grades and the formulas for the variances for contained copper and gold, were used to simulate deterministic variances of 2.3302 mt² for a copper content of 51.3 mt, and 10.5970 kg² for a gold content of 78.1 kg.

In Table 4 are listed sets of twelve simulated variance estimates for copper and gold contents.

TABLE 4. Simulated variances for copper and gold contents

No.	var(Cu)	var(Au)	No.	var(Cu)	var(Au)
1	2.0269	18.1243	7	2.1693	10.1094
2	1.8163	10.9280	8	2.3151	11.7346
3	3.5187	12.4773	9	1.3230	6.2503
4	1.3867	7.0809	10	1.1189	12.9080
5	3.5203	9.7642	11	3.5414	7.2351
6	2.5627	9.7417	12	1.6856	12.3506
Average Variance				2.2487	10.7254
Deterministic Variance				2.3302	10.5970

The first step in the process of simulating the variances for copper and gold contents that are listed in Table 2 is to generate variance estimates for wet mass in mt, copper factor, gold grade in kg/mt, and moisture factor, with the formula:

$$\text{var}[x(r)] = \{\Sigma[x(o) + (\Sigma r_j - 6) * \sqrt{\text{var}(x)}]^2 - [\Sigma[x(o) + (\Sigma r_j - 6) * \sqrt{\text{var}(x)}]]^2/12\}/11$$

This formula shows that a set of twelve normally distributed random numbers is used to simulate a single variance estimate. The second step is to generate a single normally distributed random number for each variable from var[x(r)], the simulated variance, and x(o), the observed measurement, with the following formula:

$$x(r) = x(o) + (\Sigma r_i - 6) * \sqrt{\text{var}[x(r)]}$$

Simulated variables and variances are then used to calculate the variances for copper and gold contents. This process was repeated twelve times, and the simulated variances for contained metals are listed in Table 4.

Fisher's F-test is applied to verify that the calculated variances and the averages of simulated variances are compatible. F-ratios of: 2.3302/2.2487 = 1.04 for copper, and: 10.7254/10.5970 = 1.01 for gold, are so close to unity that calculated and simulated variances are statistically identical.

The question remains whether the sets of simulated variances for copper and gold contents in Table 4 are homogeneous, too. Bartlett's chi-squared test is applied to each set to check whether simulated variances are homogeneous, and thus whether the applied simulation model is valid. The chi-squared test for homogeneity of variances is based on the following formula:

$$x^2 = \Sigma(k_j - 1) * \ln[\text{var}(\bar{x})] - \Sigma\{(k_j - 1) * \ln[\text{var}(x_j)]\}$$

in which

x^2 = calculated chi-squared value
$\ln[\text{var}(x_j)]$ = natural logarithm for jth variance
$\ln[\text{var}(\bar{x})]$ = natural logarithm for average variance
k_j = number of measurements in jth set

A comparison of calculated chi-squared values with tabulated values from the chi-squared distribution at 11 degrees of freedom reveals that the probability of 9.264 for copper falls between 8.148 at 70% and 10.341 at 50%, and that the probability of 5.359 for gold falls between 4.575 at 95% and 5.578 at 90%. Probability levels for calculated chi-squared values are neither too low nor too high so that there is no evidence that the simulation model is invalid.

Possible effects of periodicities in the "@RAND" function can be assessed by increasing the number of normally distributed random numbers in each set to match the number required for a particular simulation model. In the case that a set of variances for large sequences of normally distributed random numbers is homogeneous, the standard uniform distribution of pseudo-

and mill feed are expected to vary independently which implies that these variables are not correlated, and that covariances need not be taken into account. In the case that the stickiness of crushed ore is a function of its moisture content, the precision of the belt scale, and thus the variance of wet mass, could become a function of moisture content. Under such conditions stickiness would not only result in a higher variance for the wet mass of crushed ore that is transferred onto a heap for leaching or into a plant for processing but could also cause a bias or systematic error.

The precious metal content in kilograms of a quantity of ore or concentrate is calculated from the wet mass in metric tons, the precious metal grade in kg/mt, and the moisture content in percent. In formula:

$Me = Mw * a * MF$

in which:

Me = precious metal content in kg
Mw = wet mass in mt
a = metal grade in kg/mt
MF = moisture factor: (100 - % H_2O)/100

For example, a wet mass of 25 000 mt was measured with a belt scale for which a CV of 2.1% was reported. Its moisture content of 2.4% and gold grade of 3.2 g/mt were measured with a precision of 4.5% respectively 3.6% in terms of a coefficient of variation. Based on these coefficients of variation the variance for wet mass is: $(25{,}000 * 2.1/100)^2 = 275{,}625$ mt^2, the variance for moisture content is: $(2.4 * 4.5/100)^2 = 0.011{,}664\%^2$, and the variance for gold grade is: $(3.2 * 3.6/100)^2 = 0.013{,}271$ (g/mt)^2.

Substituting variables and variances in the mass, grade and moisture terms of the variance for a gold content of: (25 000 * 3.2/1 000) * (100 − 2.4)/100 = 78.08 kg results in the following components:

Wet mass	: $78.08^2 * 275{,}625/25{,}000^2$	= 2.6886
Gold grade	: $78.08^2 * 0.013{,}271 * 10^{-6}/0.0032^2$	= 7.9010
Moisture content	: $78.08^2 * 0.011{,}664 * 10^{-4}/0.976^2$	= 0.0075
Gold content	:	= 10.5970

The factor 10^{-6} in the grade term accounts for the use of a gold grade in kg/mt rather than g/mt when calculating the gold content in kg.

In Table 2 are listed various precision estimates that are based on the gold content of 78.1 kg and its variance of 10.5970 kg^2.

These terms show that the measurement of wet mass contributes: 2.6886 * 100/10.5970 = 25% to the variance for contained gold, that the measurement of metal grade adds: 7.9010 * 100/10.5970 = 75%, and that measurement of moisture content accounts for: 0.0075 * 100/10.5970 = 0.1% only. Hence, the variance for this gold content of 78.1 kg can be reduced, and its precision improved most of all, by reducing the total variance for grade.

The total variance for the measurement of gold in its matrix can be optimized most effectively if the sampling variance and the variances of preparation and assaying were known. Due to the single particle or nugget effect the variance of assaying is often the largest component of the total variance. The analytical variance can be reduced by screening at 100 - 150 mesh each test sample that contains coarse gold, by assaying coarse and fine fractions separately, and by reporting their weighted average as the most reliable grade estimate. The question whether weighted averages for test samples are unbiased estimates for gold in its original matrix is intriguing.

Mechanical sampling systems should preferably be designed to collect pairs of interpenetrating gross samples. A mechanical sampling system should be tested for bias before measurements on final system samples are accepted as unbiased estimates for metal grades. Even without a mechanical sampling system pairs of interpenetrating gross samples should be collected from time to time to obtained a reliable estimate for the total variance of the metal grade, and to ensure that the manual sampling procedure is optimized. After all, a simulation model requires reliable variance estimates for the most dominant variables, and realistic variance estimates for all others.

TABLE 2. Precision estimates for contained gold

Parameter	Symbol	Value
Gold in kg	Au	78.1
Variance in kg^2	var (Au)	10.5970
Standard deviation in kg	sd (Au)	3.2553
95% Confidence interval *	95% CI	
in kg		± 6.5
in %		± 8.3
95% Confidence range in kg	95% CR	
Low	Au − 95% CI	71.6
High	Au + 95% CI	84.6

*based on : z0.95 * sd (Au)

Normally Distributed Random Numbers

Spreadsheets provide pseudo-random number generators that are based on a standard uniform distribution within an interval of: $0 \leq ri \leq 1$. The term "pseudo-random" implies that the same seed will always generate the same sequence of random numbers. More advanced random number generators are reseeded with an internal variable from the microprocessor each time the generator routine is called to ensure that the probability for periodicities to occur is either eliminated or reduced to a minimum.

The Central Limit Theorem implies that the sum of n identically distributed independent random variables approximates a normal distribution with a mean of $n\mu$ and a variance of $n\sigma^2$ where μ and σ^2 are the mean and the variance of the population. If the variables $x_1, x_2, \ldots, xn$, follow the standard uniform distribution, then $\mu = 0.5$ and $\sigma^2 = 1/12$. Hence the sum of a set of n random numbers from the standard uniform distribution approximates a normal distribution with a mean of 0.5n and a variance of n/12.

The choice of n in the process of generating normally distributed random numbers is essentially a matter of computing efficiency. The larger the number of uniformly distributed random numbers in a set, the more closely the mean approaches the normal distribution. A set of twelve standard uniform random numbers provides a realistic approximation of the normal distribution, and eliminates the need to transform from nonstandard normal to standard normal by dividing the variance by the factor 12.

Based on these considerations the following formula is an efficient choice to generate the variances and variables for the function that describes their interaction in a mineral processing plant:

$$x(r) = x(o) + \sqrt{var(x)} * \left(\sum_{j=1}^{j=12} r_j - 6\right)$$

in which:

x (r) = simulated variable
x (o) = observed measurement
var (x) = variance estimate for x (o)
r_j = jth standard uniform random number

A simple example will be used to demonstrate how to generate normally distributed random numbers. The sum of twelve standard uniform random numbers that were generated in sequence with the "@RAND" function in a spreadsheet turned out to be 4.9123. If a wet mass of 25 000 mt were measured with a variance of 275 625 mt^2, a normally distributed random number for this wet mass would be: $25\,000 + \sqrt{275\,625} * (4.9123 - 6) = 25\,000 + 525 * (-1.0877) = 24\,429$ mt. Another sum of twelve standard uniform random numbers was 6.1462 which results in a normally distributed random number of: 25 000 + 525 *

TABLE 1. Precision estimates for contained copper

Parameter	Symbol	Value
Copper in mt	Cu	51.3
Variance in mt²	var (Cu)	2.3302
Standard deviation in mt	sd (Cu)	1.5265
95% Confidence interval *	95% CI	
in mt		± 3.05
in %		± 6.0
95% Confidence range in mt	95% CR	
Low	Cu − 95% CI	48.3
High	Cu + 95% CI	54.4

*based on : z0.95 * sd (Cu)

cause much uncertainty in inventories. A simulation model can be applied to assess how random variations impact on the precision of concentrate inventories.

Variances for Functions

A simulation model is based on a set of variables that each displays a degree of variability for which the variance is the basic measure. All variables that interact in a mineral processing plant are continuous but constrained within probabilistic limits. Unbiased, or at least realistic variance estimates for the wet mass, moisture content and metal grades of mill feed, and for the metal grades of concentrate and tailings are needed to compute variances estimates for dry mass, metal content and percent recovery.

The variance for a set of stochastic variables in a function interact in a deterministic model that finds its origin in calculus, and more particularly in partial derivatives of multivariate functions (5). The variance for a general function is the sum of the squared partial derivative for each variable in the set multiplied by its variance so that the following formula applies:

$$\sigma^2(y) = \left(\frac{\partial y}{\partial x_1}\right)^2 \sigma^2(x_1) + \left(\frac{\partial y}{\partial x_2}\right)^2 \sigma^2(x_2) + \ldots + \left(\frac{\partial y}{\partial x_n}\right)^2 \sigma^2(x_n)$$

This formula is applicable to all functions. Sigma symbols imply unknown population variances which, in practical applications, are replaced with variance estimates from samples. The set of variables in a function should be statistically independent. Otherwise, the effect of correlations should be taken into account by conditional simulation. Sets of variables can be tested for statistical dependencies by applying correlation-regression analysis.

For complex systems such as a mineral processing plant with a heavy medium section, and flotation circuits and thickeners for different concentrates, the partial derivatives that deterministic models require are difficult to develop. In fact, complex systems are the very reason why simulation models have become such a powerful tool. For the disadvantage of deteriministic models is that variances become constants. A simple function will be used to show how to simulate the variances and variables, and how to check the validity of the simulation model.

The metal content in metric tons of a quantity of ore or mineral concentrate is calculated from the wet mass in metric tons, and its metal grade and moisture content in percent with the following formula:

$$Me = Mw * AF * MF$$

in which:
Me = metal content in mt
Mw = wet mass in mt
AF = metal factor: %a/100
%a = metal grade in percent
MF = moisture factor: (100 - $\%H_2O$)/100
$\%H_2O$ = percent moisture

Based on the formula for the variance of a multivariate function the partial derivatives for this function are:

$$\left(\frac{\partial Me}{\partial Mw}\right) = AF * MF \left(\frac{\partial Me}{\partial AF}\right) = Mw * MF \left(\frac{\partial Me}{\partial MF}\right) = Mw * AF$$

Substituting variables and variances in the formula for the variance of metal content results in the following terms:

$$\text{var}(Me) = (AF * MF)^2 \text{ var } (Mw) + (Mw * MF)^2 \text{ var } (AF) + (Mw * AF)^2 \text{ var } (MF)$$

Substituting σ^2 with var(x) implies that unknown population variances are replaced with variance estimates that are measured in samples. The formula for var(Me) can be further simplified by multiplying the term for the wet mass with $(Mw/Mw)^2$, the term for the grade factor with $(AF/AF)^2$, and the term for the moisture factor with $(MF/MF)^2$, and then multiplying the sum of all terms with $(Me/Me)^2$.

Following is the simplifed formula to calculate the variance for contained metal or metal content:

$$\text{var}(Me) = Me^2[\text{var}(Mw)/Mw^2 + \text{var}(AF)/AF^2 + \text{var } (MF)/MF^2]$$

A numerical example shows how the variance for metal contents or contained metal is calculated. A wet mass of 10 000 mt was measured with a belt scale for which a coefficient of variation (CV in %) of 1.6% was reported. In terms of coefficientss of variation the variances of sampling, preparation and analysis for a copper grade of 0.53% and a moisture content of 3.2% were 2.5% and 5.0% respectively.

The coefficient of variation is equivalent to the standard deviation as a percentage of the measured variable so that the variance for a wet mass of 10 000 mt is: $(10\,000 * 1.6/100)^2 = 25\,600$ mt². Similarly, for a copper grade of 0.53% the total variance, which is the sum of the variances for sampling, sample preparation and assaying, is: $(0.53 * 2.5/100)^2 = 0.000,176\%^2$, and the total variance for a moisture content of 3.2% is: $(3.2 * 5.0/100)^2 = 0.0256\%^2$.

Based on these variances the terms of the variance for a copper content of: 10 000 * (0.53/100) * (100 - 3.2)/100 = 51.304 mt are:

Wet mass	: $51.304^2 * 25{,}600/10{,}000^2$	= 0.6738
Copper grade	: $51.304^2 * 0.0001{,}176 * 10^{-4}/0.0053^2$	= 1.6492
Moisture content	: $51.304^2 * 0.0256 * 10^{-4}/0.968^2$	= 0.0072
Copper content	:	= 2.3302

Multiplication with 10^{-4} accounts for the fact that metal and moisture factors rather than percentages are used to calculate their contributions to var(Cu), the variance for contained copper. In Table 1 are listed precision estimates for copper content.

The factor z0.95 = 1.96, which is valid for a symmetrical 95% probability of the Gaussian or normal distribution, is usually rounded to 2.0.

These terms indicate that the measurement of wet mass contributes: 0.6738 * 100/2.3302 = 29% to the variance for contained copper, that the measurement of grade accounts for: 1.6492 * 100/2.3302 = 71%, and that the measurement of moisture adds only: 0.0072 * 100/2.3302 = 0.3%. Hence, the precision for contained copper can be improved most of all by reducing the variance for grade. If the total variance were partitioned into the sampling variance, were partitioned into the sampling variance, the preparation variance and the analytical variance, then the precision for the measurement of copper in crushed ore can be optimized most effectively.

The wet mass, metal grade and moisture content of mined ore

in a set. The theorem also underlies the general sampling formula which is based on the additive property of variances with identical dimensions.

The following formula is a simplified version of the general sampling formula as it applies to a single sampling unit such as a shift's production:

$$var(t) = var(s)/n + var(p) + var(a)/k$$

In which:

var(t) = total variance for a sampling unit
var(s) = sampling variance
var(p) = preparation variance
var(a) = analytical variance
n = number of increments/sampling unit
k = number of measurements/test sample

Simulation models for mineral processing plants should take into account the effect of a serial correlation on the sampling variance[1,4]. Measurements with an on-stream analyzer in particular can be used to compute the terms of time series variances for each mass flow that is interrogated. The term that is closest to the time interval between mechanically collected increments is a reliable estimate of the sampling variance for the grade of this mass flow.

A significant serial correlation exists if the F-ratio between the variance for randomized on-stream measurements and the first term of the time series variances for the ordered measurements exceeds the tabulated value at 95% or 99% probability with the appropriate degrees of freedom. In the case that the calculated F-ratio is below the tabulated value the measurements in the set are randomly distributed, and thus statistically independent. Often, the sampling variance is significantly lower than the sum of the variances of sample preparation and assaying so that the serial correlation impacts only marginally and not significantly on the total variance. Nevertheless, the preparation and analytical variances are extrinsic to the variability of a metal grade so that only the sampling variance is required for simulation models.

Sets of on-stream measurements for mass flows in mineral processing plants invariably display significant serial correlations. Time series variances for on-stream measurements are computed with the following formula:

$$var_i(x) = \frac{\Sigma[x_{(j+i)} - x_j]^2}{2n - 1}$$

in which:

$var_i(x)$ = variance at ith spacing
$x_{(j+i)}$ = (j + i) th measurement
x_j = jth measurement
i = spacing between measurements
n = number of measurements in the set

A sampling viariogram is a graph in which a set of time series variances is plotted against their spacings[3,4]. If a significant serial correlation exists, then the term of the time series variances that matches the interval between mechanically collected increments is the most reliable estimate for the sampling variance. Dividing it by the number of on-stream measurements in a shift, and adding the variances of preparation and assaying for a sample from the slurry that is measured with the on-stream analyzer, generates a reliable estimate for the total variance of its metal grade. This variance also plays an important role in calibration procedures for on-stream analyzers.

Generally, the total variance for the moisture content or metal grade of a sampling unit can be estimated by dividing the sampling variance by the number of primary increments in the set that constitutes a gross sample, and then adding the variance of preparation and assaying. The preparation variance can be estimated from duplicate test samples, and the analytical variance from assays of duplicate tests portions, with the following formula:

$$var(x) = (\pi/4) * [\Sigma (| x_{1j} - x_{2j} |)/k]^2$$

in which:

var(x) = variance for a single measurement
x_{1j} = first measurement on jth pair
x_{2j} = second measurement on jth pair
k = number of measurements in the set

A cost effective method to estimate the total variance for the metal grade of a mass flow is to collect a pair of interpenetrating gross samples from a set of no less than four sampling units. A major advantage of this sampling regime is that it takes into account, almost quantitatively, the effect of a serial correlation on the sampling variance[3,4]. For the sampling variance of each interpenetrating gross sample is, in effect, equal to the seond term of the time series variances rather than the first term.

Ideally, the analytical variance should be the dominant component in the process of sampling, preparation and assaying. For coarse materials such as crushed ore the sum of the sampling and preparation variances often exceeds the analytical variance. For fine materials such as ball mill discharge, cyclone overflow, tailing and concentrate the sampling variance need not be more than a fraction of the analytical variance. Moreover, the variance for preparing a test sample can easily be reduced to match the variance for assaying a test portion. Rod mill discharge is difficult to sample manually, and expensive to sample mechanically, so that unbiased measurements with acceptable precision characteristics are hard to obtain.

Concentrates in thickeners contribute a large measure of uncertainty to production and inventory, and thus large variance components to simulation models for mineral processing plants. Measuring the wet mass of concentrate with a static scale after dewatering or drying reduces the variance for the wet mass of concentrate production to the lowest possible level.

Mineral concentrates can be sampled for moisture content and metal grades with a high degree of precision. In fact, sampling and weighing concentrate after dewatering or drying at a mineral processing plant generates the first point of reference against which ore reserves can be reconciled, and smelter returns can be compared most effectively.

The variance for the wet mass of mill feed that is measured with a belt scale can be estimated at low cost. A monitor program that retains a running database of differences between a belt scale's set point under applied load upon completion of calibration and the set point observed under applied load prior to the next calibration is an effective technique to estimate the variance for wet mass. During a maintenance period a conveyor with the belt scale could also be operated, with a chain or a static load applied, for intervals of 30 - 60 minutes. After each interval the observed mass is recorded, and the test is repeated until no less than four observations are obtained. Due to its short time base the variance for this test is lower than the variance for the monitor program.

Calibration data for a static scale provide sufficient information to estimate the variances at gross and tare loads, and thus the variance for the the net wet mass of concentrate in a single rail car, truck or bulk bag. The sum of the variances for any set of units becomes the variance for the cumulative wet mass that was loaded during a production period. This variance can also be used in simulation models.

Belt scales are perfectly acceptable to measure the wet mass of crushed ore at a mineral processing plant or a heap-leach operation but not precise enough to measure the wet mass of concentrate during transfer to inventory at a mine. In particular if large inventories are retained for long periods of time, all random variations and systematic errors in mass measurements accumulate and

random numbers that the "@RAND" function provides is also reliable for simulation models that require large sequences. However, if the periodicities in the "@RAND" function and the required sequences were to coincide, the random number generator should not be applied to process simulations.

Summary

Spreadsheet software can be used to develop advanced simulation models for mineral processing plants . The pseudo-random number generator for the standard uniform distribution that spreadsheet software routinely provides should be tested to check its performance and suitability for simulation models. Statistical techniques are applied to check the validity of the simulation model by testing pairs of variances for compatibililty, and sets of variances for homogeneity. Models that simulate all variables and variances that interact in a mineral processing plant are more realistic and effective than models that simulate variables from constant variances.

Reliable variance estimates are the key to meaningful simulation models for mineral processing plants. Unless the variances and variables are generated simultaneously a simulation model for a mineral processing plant cannot truly reflect its behaviour. Nor could it generate a reliable variance estimate for the mass of tailings, and, by implication, for the mass of concentrate that is transferred without weighing to a thickener, or to an inventory without weighing.

Sets of on-stream measurements can be used to compute terms of time series variances, and to estimate the sampling variance for each metal grade. Pairs of interpenetrating samples are effective to estimate the total variance for moisture and metal grades of mill feed and concentrate, and for metal grades of different mass flows in a mineral processing plant.

The effect of dependencies between variables can be taken into account by conditional simulation on the basis of correlation-regression parameters.

Metal grades in mill feed and tailings are frequently dependent so that the simulation model should reflect such a correlation. Arithmetic means of on-stream measurements of concentrate grades are often biased due to the fact that low mass flows in flotation cells cause higher than average metal grades in concentrate. A weighted average grade that is based on the sum of products of metal grades and mass flows for a shift's production is more likely to be unbiased.

Simulation can be applied to simple two-product mineral processing plants and complex plants with heavy medium sections, and flotation circuits for different concentrates alike. Simulation models provide a more profound understanding of interactions between variables and variances in a mineral processing plant than daily, weekly or monthly metallurgical balances.

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